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主　論　文　の　要　旨

論文題目　Inner Configurations in Combinatorial Designs and their Linkage to Codes and Sequences
（組合せデザインの内部構造とその符号および系列との関連性）

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論　文　内　容　の　要　旨

Combinatorial design theory typically considers incidence relations of a given set of elements to a family of subsets of the set. For instance, a fundamental class of combinatorial designs, named after Jakob Steiner, a nineteenth-century mathematician, is defined as follows:

Let t, v, k and λ be positive integers. A *Steiner t-design*, $S(t,k,v)$, is an ordered pair $(V,\mathcal{B})$, where V is a finite set of v elements called *points*, and $\mathcal{B}$ is a set of k-element subsets of V called *blocks*, such that each t-element subset of distinct elements of V is contained in exactly one block of $\mathcal{B}$. When the block size $k=3$ and $t=2$, an $S(2,3,v)$ is called a *Steiner triple system* and denoted STS(v).

Over a hundred and fifty years have passed since Steiner triple systems, which were among the first combinatorial designs systematically studied, arose from the infancy of abstract algebra and finite geometry. Interacting with surrounding branches of mathematics, studies of disjointed problems, such as existence of an STS(v) and particular algebraic structures, posed in several distinct fields of mathematics have developed into modern theory of combinatorial designs. Accordingly a sizable proportion of modern combinatorial designs had come into being with their own motivations, and hence it is of fundamental importance to study combinatorial designs themselves in purely mathematical sense.

At the same time, today's combinatorial design theory has various, close relations to areas within or even outside mathematics, say, computer science and engineering. Hence, an investigation in interacting areas may allow both combinatorial design theory and related fields to advance and extend the circumference of their sphere.

With these thoughts in mind, we discuss in this thesis several combinatorial design theoretic problems and their linkage to another field. As quintessential manifestations of the interactions with another field can be seen between partial structures of combinatorial designs and computer science, our primary focus in this thesis is on inner structures, as it is sometimes called "configurations," in combinatorial designs.

Chapter 1 is dedicated to providing a concise introduction to configurations in combinatorial designs and their application to computer science, a fairly new scientific field having brought fresh insights to combinatorics. Beginning with a brief account of some classical, fundamental combinatorial designs such as Steiner t-designs, this chapter deals with basic notions concerning configurations and their relations to so-called "frequency hopping (FH) sequences" and "X-codes."

In Chapter 2 we study special configurations in an STS and consider the following conjecture:

A (k,l)-*configuration* in an STS is a set of l blocks whose union contains precisely k points. In 1973 Paul Erdős conjectured that for $r \geq 4$ there is an integer $v_0(r)$ such that for every $v > v_0(r)$, $v \equiv 1,3 \pmod 6$, there exists a Steiner triple system on v elements containing no $(k+2,k)$-configurations for every $1 < k \leq r$. Such an STS is said to be r-*sparse*.

His conjecture, sometimes called the "r-sparse conjecture," was proved to be true only for the case when $r = 4$. Whilst sparseness is now known to have various application fields, less is known about higher sparseness. In fact, No example of r-sparse systems is realized for $r \geq 7$ (and $v > 3$), and no affirmative answer to the r-sparse conjecture is known in this range.

Throughout the chapter we revolve around the r-sparse conjecture from its origin to the current state. Among others, we show that for every positive $v \equiv 1, 19 \pmod{54}$ there exists a 5-sparse STS(v). We also prove that for $r \geq 13$ there exists no r-sparse STS admitting an abelian group, acting transitively on the point set, as a subgroup of the full automorphism group. An analogous problem is also considered for "4-cycle systems," which are fairly new combinatorial designs.

While the r-sparse conjecture is a problem on Steiner triple systems, we take a look in Chapter 3 at another kind of configurations in Steiner 2-designs.

An $S(2,t,v)$ is said to be *halvable* if its block set $\mathcal{B}$ is partitioned into two isomorphic configurations $\mathcal{B}_1$ and $\mathcal{B}_2$. Obvious necessary condition for the existence of a halvable $S(2,k,v)$ is that the number $|\mathcal{B}|$ of blocks is even.

While not much was known about existence of a halvable $S(2,k,v)$ even for the case $k = 3$, we present an asymptotic solution for the existence problem of several special cases. The main result on halving problem is that for any $k \leq 5$ or any Mersenne prime k, there is a constant number v_0 such that if $v > v_0$ and v satisfies the necessary conditions for the existence of an $S(2, k, v)$ with an even number of blocks, then there exists a halvable $S(2, k, v)$. We also give some construction methods of halvable $S(2, k, v)$s and give new infinite series of halvable Steiner 2-designs.

Chapter 4, the final chapter, is dedicated to studying applications of configurations to computer science through investigations into two typical applications of configurations to codes and sequences.

The first half of the final chapter deals with a relation of configurations to particular families of FH sequences for "FHMA spread-spectrum communications." Generalizing the known equivalence relation between an FH sequence and a particular combinatorial design, called a "partition-type difference packing," we present a combinatorial construction method of FH sequences and provide infinitely many new optimal families with respect to the so-called Lempel-Greenberger bound.

In the latter half, we study relations of specific configurations, including partly those considered in the r-sparse conjecture, to a speacial class of codes employed in "X-tolerant test response compaction techniques" for integrated circuits, namely X-codes. We formulate a combinatorial model of the X-tolerant compaction and give fundamentals to study the mathematical aspects of the compaction technique. We also present a construction method of X-codes through an investigation into combinatorial designs and their inner configurations such as 4-sparse STSs. A new problem in combinatorial design theory brought by the X-compaction is also discussed.